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ABSTRACT

The aim of this study is to determine the overall effect of multiple representation based instruction on mathematical achievement. Meta-analysis method was used for this aim. The sample of the study consisted of 33 experimental studies within 10 publications which were selected according to certain criteria. In data analysis, mean effect size of the selected studies was examined. The effect size distribution of the selected studies demonstrated a heterogeneous structure which led to a preference for random effects model (Q=66.320; p=.000). The data of the current study were analyzed using Comprehensive Meta-Analysis (CMA), which is a specialized statistical software. The result of the Z-test revealed a statistically significant effect size (Z=7.015; p<.05). In order to find evidence for reliability, both a graphic approach (funnel plot) and Orwin’s fail-safe N method were used to assess the potential publication bias. Findings of these tests suggested no bias in the data. As a result, the current investigation revealed that there was an overall medium and positive relationship between multiple representation based instruction and mathematical achievement.

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Keywords: Mathematical achievement, mathematics instruction, meta-analysis, multiple representation based instruction

INTRODUCTION

Some methods used in mathematics education are important in terms of conceptual learning and mathematical achievement. One popular method used is multiple representations based instruction (Ainsworth, 2008). This method is encountered more frequently with the evolution and widespread use of technology in educational settings. The number and quality of representation types for mathematical concepts have both increased with more effective and efficient use of educational technology (İpek & Baran, 2011).

Multiple representation based instruction was introduced by Lesh “Lesh Multiple Representations Translations Model (LMRTM). Several representation types were used in this study such as pictures, physical models, manipulative models, real-world situations, metaphors, oral language and written symbols. Representations are very important in understanding mathematical concepts. According to this model, if a student understood a mathematical idea then she should have the competency to relate several representations of the same idea (Lesh, Post, & Behr, 1987). The LMRTM model is illustrated in Figure 1.

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In multiple representations based mathematics instruction; manipulatives, real-world situations, oral explanations and statical visuals should be used rather than using mathematical symbols only. In some recent studies on “representation” concept which is the starting point of multiple representations based instruction; it was suggested that the skills to choose and create representations would be more important than calculation in the near future and that the students with an awareness of multiple representations would also be developed in metacognition skills (Ainsworth, 2006; Kaput, 1998). In addition, some authors contended that visual representations were important in understanding mathematics and solving verbal problems (Abdullah, Halim, & Zakaria, 2014; Nash, 2012). Also, it is stated that, teachers should adjust between mathematics lessons, teaching model used and the attention to multiple ways owned by each student, because each student character in the learning process has an effect on student achievement (Nugroho, Budiyono & Slameti 2019).

Many studies highlighted the fact that multiple representations based mathematics instruction enhanced understanding abstract mathematical concepts, provided meaningful mathematical understanding and contributed in students’ conceptual construction of mathematical knowledge (Ainsworth, 2006; Dreher & Kuntze, 2015; Goldin, 2004; NCTM, 2008). In other words, students realize the (un)necessary details related or some properties specific to a topic and will be able to concretize the concepts by reconstructing them in their minds. Such deeper learning offers another reason why multiple representations are so important (Ainsworth, Bibby & Wood 1997; İzgiol, 2014).

Many studies in mathematics education have focused on multiple representation skills. These studies investigated the role of representations in understanding and teaching mathematics, shifting between representation types, representation awareness in students, use of representations, representation preferences and representation in computer aided instruction with the increasing use of technology in educational settings (Adu-Gyamfi, 2007; Akkuş Çikla, 2004; Bal, 2014; Dreher & Kuntze, 2015; Duncan, 2010; Durmuş & Yaman, 2002; Gilbert, 2010; Kendal, 2002; Mallet, 2007). Several studies in the literature reveal findings regarding the development in students’ mathematical skills and performances by using technology assisted multiple representation applications (Adıgüzel & Akpınar, 2004; Çetin, 2016; İzgiol, 2014; Kaya, 2015). On the other hand, some multiple representation based instruction studies have focused on different topics of mathematics. Some examples of this type including those investigating the effect of using multiple representations (symbolical, graphical, tables and verbal) in teaching functions on mathematical achievement (Akkoç, 2006; Can, 2014; Garay, 2001), the effect of using multiple representation based algebra instruction on achievement in mathematics and algebra (Akkus & Cakiroğlu, 2010; Gürbüz & Şahin, 2015; İzgiol, 2014; Kaya, 2015; Ozgun-Koca, 2001; Yakar & Yılmaz, 2017), the effect of multiple representation based number instruction on achievement (Çetin, 2016; Sun, 2006), the effect of teaching derivative with multiple representation (Goerdt, 2007), examining multi-representations on knowledge of function (Rider, 2004) and the effect of multiple representation in the context of problem solving (Adıgüzel & Akpınar, 2004; Hwang, Chen, Dung, & Yang, 2007) have all pointed out to a need to conduct this meta-analysis.

The relevant literature seems to lack a grounded theory for the overall effect of multiple representation based instruction on mathematical achievement which indicates a strong need for a meta-analysis. No general and strong argument was made regarding the positive relationship between these variables because multiple representation based instruction was shown to have conflicting effects on different subdimensions of mathematical achievement ranging from negative effects to low or high positive effects. Therefore, this study aimed at finding a general relationship using a mathematically robust method. Meta-analysis is such a powerful statistical method in which the effect size of a variable on another variable can be predicted by using a number of alike or different scientific findings of studies regarding the same
variables (Ellis, 2010). This method is also known as analysis of the analyses (Welkowitz, Cohen, & Lea, 2011) and yields a general and convincing argument.

In light of the inconsistent findings on the multiple representation based instruction-mathematical achievement relationship published recently, the aim of the current study is to quantitatively synthesize these studies for the purpose of providing an overall view on this relationship. For this aim, this paper attempts to address the following research question: “What is the overall correlation between multiple representation based instruction and mathematical achievement?”

**METHOD**

In this paper, research conducted on the effect of multiple representation based instruction on mathematical achievement were synthesized using meta-analysis to examine the relationship between the two variables. We saw that researchers studied the effective of multiple representation instruction on achievement with experimental method on different groups, generally. By this way, researchers found out many different results about effectiveness. Researching the effect of methods with experimental studies is important for learning the success of mathematics (Mulyanto, Gunarhadi & Indriayu, 2018). But, the main question of this study is, what the real effectiveness level of multiple representation based instruction is on mathematical achievement. So that, to examine the effectiveness of multiple representation based instruction on mathematical achievement, the meta-analysis method was employed.

Meta-analysis is a statistical method of synthesizing data. One use of this method is to compare and reveal the effect of a certain treatment on an outcome variable (Borenstein, Hedges, Higgins, & Rothstein, 2011).

The procedures followed in this study were as; selection of the variables, literature review, establishing inclusion/exclusion criteria, identification of themes, formulating research questions, coding, analysis, calculation of effect sizes, conducting test of heterogeneity, selection of analysis model, calculation of overall effect and interpretation (Dinçer, 2014).

**Collecting Data**

The sample of this study consisted of articles and dissertations on the effect of multiple representation based instruction on mathematical achievement. These publications included mostly those written in English as well as those in Turkish.

For this aim, Proquest Dissertations and Theses, Google Scholar and online Dissertation and Thesis center of Turkish Higher Education Council (YÖK Tez Merkezi) databases were queried using the keywords of “multiple representation based mathematics instruction”, “multiple representation”, “mathematics education”. These searches yielded 1 article, 7 dissertations and 2 master’s theses. Some of these 10 publications included subsections and experimental results in these subsections which added up to 33 different experimental studies. These 33 different experimental findings in 10 publications were synthesized using meta-analysis method. In these publications, sub-dimensions of mathematical achievement such as verbal problem solving, reading graphs, interpreting graphs, reading tables, interpreting tables, solving equations, building equations, achievement in algebra, achievement in shifting between representations and algebraic reasoning found in 33 different experimental findings were included in the analysis as a single outcome variable as mathematical achievement.

A publication (or a subsection of a publication) had to satisfy the following criteria to be included in the study:

1. To be accessible in the selected databases as fulltext and be downloadable in pdf format.
2. To be using a pretest-posttest experimental study design investigating the effect of multiple representation based instruction on any type and subdimension of mathematical achievement.
3. To be reporting sample sizes, means and standard deviations of experiment and control groups, t and p values of its analysis.

There are not many papers on the research question therefore all the relevant publications which added up to 10 papers overarching 33 different experimental subsections (findings) were included in the analysis. Those studies investigating the same variables were excluded if they were not experimental, did not use pretests, had quasi or weak experimental designs. Moreover, those studies using non-parametric tests such as $\chi^2$, Kruskal Wallis or Mann whitney U were also excluded from the analysis.

**Coding Process and Coding Reliability**

First of all, the identity information of the selected publications were noted and classified. These information are authors, publication year, sample features, the number of subdimensions of the dependent variable and the descriptives of these studies (sample size, mean, standard deviation, t and p values). These data are recorded by the researchers using a coding form.

Coding reliability is an important issue in meta-analysis (Bakioğlu & Özcan, 2016). Therefore, it is recommended to evaluate the data with at least two researchers (Açıkel, 2009). Two researchers worked in this study. They discussed the fit of some publications to this study and then identified the final list of appropriate publications with negotiations. The coding reliability coefficient calculated with a formula (Miles, Huberman, & Huberman, 1994) between these two coders was found as $86\%$ ($\frac{33}{33+5} \times 100\%$).

**Validity and Reliability of the Study**

In order to ensure content validity, all studies on the effect of multiple representation based instruction on mathematical achievement were queried. In order to ensure reliability, publication bias was examined (Schmidt & Hunter, 2014). To examine publication bias, funnel plot and Orwin’s fail-safe N method were used. Fail safe N is a reliability measure in meta-analysis. It represents the number of unpublished studies with zero effect sizes that would be sufficient to bring the observed statistically significant overall effect size down to the level of being statistically non-significant (Bakioğlu & Özcan, 2016).

**Data Analysis**

In order to find the overall effect size, meta-analysis method was used in this study. In this method, it is aimed to calculate the difference between the experiment and control group means of the dependent variable (achievement, verbal problem solving, reading graphs, interpreting graphs, reading tables, interpreting tables, solving equations, building equations, achievement in algebra, achievement in shifting between representations and algebraic reasoning) by using these means as they cannot be obtained from the same scale (Schmidt & Hunter, 2014). In the analysis; sample sizes, means and standard deviations were used for most of the publications whereas only means and p values were used for two publications. A specialized statistical software CMA (Comprehensive Meta-analysis) was used to conduct the analysis. In order to find the overall effect, mean effect size was calculated and the significance level of the analysis was found as 95%.
FINDINGS

In this study, the research question “What is the effect of multiple representation based instruction on mathematical achievement?” was investigated and 10 publications selected according to the identified inclusion criteria were reviewed. In this review 33 different experimental findings regarding the research question were analyzed. The significance level of the analysis was taken as p=0.05. The characteristical information of the publications included in the analysis were given in Table 1. In the table, the authors and publication years of the publications were demonstrated along with effect sizes. In addition, the contents of the publications, i.e. the topic, education level, sample size and publication types were shown.

Table 1. Features of the selected publications

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Effect Size</th>
<th>Education Level</th>
<th>Sample Size</th>
<th>Publication Type</th>
<th>Database</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garay Jose Av.</td>
<td>2001</td>
<td>.047</td>
<td>University</td>
<td>29 – 23</td>
<td>Dissert.</td>
<td>Proquest</td>
</tr>
<tr>
<td>Adiguzel &amp; Akpinar</td>
<td>2004</td>
<td>.318</td>
<td>Middle School</td>
<td>17 – 10</td>
<td>Article</td>
<td>Google Scholar</td>
</tr>
<tr>
<td>Goerdt Lee</td>
<td>2007</td>
<td>.536</td>
<td>University</td>
<td>29 – 22</td>
<td>Dissert.</td>
<td>Proquest</td>
</tr>
<tr>
<td>Kaya-a</td>
<td>2015</td>
<td>.588</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Kaya-b</td>
<td>2015</td>
<td>.960</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Kaya-c</td>
<td>2015</td>
<td>.335</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Kaya-d</td>
<td>2015</td>
<td>.649</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Kaya-e</td>
<td>2015</td>
<td>.726</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Kaya-f</td>
<td>2015</td>
<td>.800</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Kaya-g</td>
<td>2015</td>
<td>.439</td>
<td>Middle School</td>
<td>30 – 30</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Author</td>
<td>2016</td>
<td>.952</td>
<td>Middle School</td>
<td>27 – 27</td>
<td>Dissert.</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Rider Lyn</td>
<td>2004</td>
<td>1.271</td>
<td>University</td>
<td>100 – 213</td>
<td>Dissert.</td>
<td>Proquest</td>
</tr>
<tr>
<td>İzgiol</td>
<td>2014</td>
<td>.278</td>
<td>University</td>
<td>35 – 38</td>
<td>Thesis</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Can</td>
<td>2014</td>
<td>1.316</td>
<td>Secondary</td>
<td>29 – 26</td>
<td>Thesis</td>
<td>YÖK tez</td>
</tr>
<tr>
<td>Özgünkoca-c</td>
<td>2001</td>
<td>-.294</td>
<td>Secondary</td>
<td>10 – 5</td>
<td>Dissert.</td>
<td>Proquest</td>
</tr>
<tr>
<td>Özgünkoca-e</td>
<td>2001</td>
<td>.000</td>
<td>Secondary</td>
<td>10 – 5</td>
<td>Dissert.</td>
<td>Proquest</td>
</tr>
<tr>
<td>Özgünkoca-g</td>
<td>2001</td>
<td>.208</td>
<td>Secondary</td>
<td>10 – 5</td>
<td>Dissert.</td>
<td>Proquest</td>
</tr>
</tbody>
</table>
The sample is total 1678 which consisted of the total of experiment groups (N=834; % 49.71) and the total of control groups (N= 844; % 50.29) as shown in Table 1. Among the experimental findings (publications or subsections of publications) included in the analysis one was obtained from an article, 30 from dissertations and 2 from theses. The effect sizes of the studies included in the analysis varied between –.551 and 1,316. Three of the studies were found to have negative effect sizes. Negative effect sizes demonstrate an effect in favor of control groups. In 30 of the included studies, representation based instruction was found to have a positive effect on mathematical achievement in favor of experiment groups. When the effect sizes of these studies are examined, 8 studies had strong, 6 studies had medium and 19 studies had low positive effects. Most of the included studies targeted middle (n=12; 36,3 %) and secondary school (n=17; 51,5%) students.

The studies included were tested for heterogeneity to identify the appropriate model for analysis. The test of heterogeneity indicated that random effects model was appropriate. The mean effect sizes, heterogeneous distribution value and confidence intervals are given in Table 2.

Table 2. The Findings regarding the Effect Size according to Random Effects Model

<table>
<thead>
<tr>
<th>Std. Dif. in Means</th>
<th>N</th>
<th>Standart Error (SE)</th>
<th>Z</th>
<th>P</th>
<th>%95 Confidence Interval for Effect Size (ES (%95 CI))</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.721</td>
<td>.083</td>
<td>7.015</td>
<td>.000</td>
<td>.420 -.746</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Q=66.320; p=.000

As shown in Table 2, the selected studies were found heterogeneous (p<.05). This means these studies are not similar. Since the effect sizes of the included studies demonstrate a heterogeneous distribution, random effects model was used (Q=66,320; p=.000). Using this model, overall effect size was found as .721 with a standard error of .083. This result suggested that multiple representation based instruction had a medium positive effect on mathematical achievement (Lipsey & Wilson, 2001). The minimum value of effect size was found as .420 and the maximum value as .746 within 95 % confidence interval. This effect size can be accepted as statistically significant (Z=7,015; p<.05).

In order to test publication bias in the analysis, funnel plot shown in Figure 2 was examined.
Figure 2. Effect Sizes’ Funnel Plot

Standart differences in means and standart errors can be observed in Figure 2. The main idea behind funnel plot is that publication bias would lead to asymmetry (Borenstein et al., 2011). Funnel plot is a scatter plot graph examining effect sizes versus sample sizes as variables, which is used to test publication bias but is not considered as sufficient on its own (Bakioğlu & Özcan, 2016).

Therefore Orwin’s fail-safe N method was used additionally and fail safe number was found as 870. According to this result from this analysis consisting of 33 experimental findings; in order to bring the observed statistically significant overall effect size down to the level of being statistically non-significant, 870 experimental studies with negative effect sizes are needed (Bakioğlu & Özcan, 2016). When only a few studies (5 – 10) are sufficient to negate the overall effect, one can concern that the mean effect might be zero (Borenstein et al., 2011). However, 870 studies are too many and overall effect size (ES=.721) was found larger than the least recommended effect size of .704 which indicated there’s no publication bias.

Forest plot graph demonstrating the effect sizes and directions of the experimental findings included in this study is shown in Figure 3.

<table>
<thead>
<tr>
<th>Study name</th>
<th>Std dev in means</th>
<th>Standard error</th>
<th>Variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
<th>Z-value</th>
<th>p-value</th>
<th>Std dev in means and 95% Cl</th>
<th>Weight (Fixed)</th>
<th>Relative weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.087</td>
<td>0.279</td>
<td>0.079</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-1.00</td>
<td>0.50</td>
<td>0.000</td>
</tr>
<tr>
<td>Value</td>
<td>-1.00</td>
<td>0.50</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Forest plot of the effect sizes of the experimental findings according to random effects model
Overall effect size was found as .721 as shown in Figure 3. According to this result, the effect of multiple representation based instruction on mathematical achievement was in favor of experimental groups. The effect sizes of the examined experimental findings ranged between -.551 and 1.316. Among 33 examined experimental findings, 30 which favored multiple representation based instruction had positive and 3 which favored traditional instruction had negative effect sizes. The publication with the largest relative weight (RW=15.60) was the work of Rider (2004). Relatively larger sample size in that work might be an explanation for this finding (Borenstein et al., 2005). On the other hand, the publication with the smallest relative weight (RW=.75) was the work of Rider (2004) about the effect of multiple representation based instruction on achievement in reading graphs.

RESULT, DISCUSSION, AND SUGGESTIONS

As a result of this meta-analysis study within 33 experimental findings (publications or subsections of publications) within 10 publications to determine the effect of multiple representation based instruction on mathematical achievement, the overall effect size was found as .721 which can be considered at medium level (Lipsey & Wilson, 2001). In addition, the 95% confidence interval values with a standard error of .083 ranged between .420 and .746. The test of heterogeneity revealed a heterogeneous distribution and the analysis was conducted using random effects model (Q=66.320; p<.05). Finally, z test showed the overall effect size was statistically significant (z=7.015; p<.01).

Based on the overall effect size found in this study, it could be suggested that multiple representation based instruction positively effected mathematical achievement. In other words, multiple representation based instruction can be argued to outperform traditional one. This finding is in line with the majority of the studies included in the meta-analysis (Adiguzel & Akpinar, 2004; Akkus & Cakiroglu, 2010; Çetin, 2016; Can, 2014; Garay, 2001; Goerdt, 2007; İzgiol, 2014; Kaya, 2015; Rider, 2004) and some other qualitative studies about the same research question (Akkoç, 2006; Gürbüz & Şahin, 2015; Yakar & Yılmaz, 2017). In contrast, this finding contradicts with the results of only a few works one of which is by Özgunkoca (2001) which demonstrated a negative effect on achievement in interpreting graphs and reading tables. Multiple representation based activities are not effective only on mathematical achievement but also enhance understanding abstract concepts and contribute in meaningful (Ainsworth, 2006; Dreher & Kuntze, 2015; Goldin, 2004; NCTM, 2008) and deeper learning (Ainsworth, Bibby & Wood, 1997; İzgiol, 2014).

Based on the findings in this meta-analysis study about the positive effect of multiple representation based instruction on sub-dimensions of mathematical achievement such as verbal problem solving, reading graphs, interpreting graphs, reading tables, interpreting tables, solving equations, building equations, achievement in algebra, achievement in shifting between representations and algebraic reasoning it can be suggested that multiple representations should be addressed at all levels of education from the primary schools to university.

A meta-analytic approach was adopted in this study therefore only experimental studies satisfying certain conditions were included. New meta-synthesis studies which might include qualitative works and use other statistical techniques can also be conducted on the same research question. Furthermore, researchers are strongly recommended to carry out future studies investigating the effect of multiple representation based instruction on variables other than achievement such as attitudes or conceptions.

Although it is still not known how many types of representation should be involved for an effective mathematics instruction, the variety in representation types will certainly make learning easier and more meaningful. In this respect, it is essential that the teachers and students be aware of possible representation types for the mathematics topic in hand.
REFERENCES


